VINITI
1983
MINISTRY OF HIGHER AND SPECIALIZED SECONDARY EDUCATION OF THE USSR
Editorial board of the journal: “Izvestiya vuzov MV i SSO SSSR”, series “Fizika”
#3872-83Dep.
UDC 530.145

A.M.Akhmeteli

MAJORANA SPINORS IN THE EQUATIONS OF CLASSICAL SPINOR ELECTRODYNAMICS

Tomsk – 1983

Introduction. In work[1], Dirac showed that in classical electrodynamics, it is practically always possible, using a suitable gauge transform, to pass to a new 4-potential of the electromagnetic field whose direction in each point will coincide with the direction of motion of a trial charge. In the context of quantum mechanics, this conclusion found a more adequate formulation in work by Schrödinger[2], who considered equations of motion for interacting complex scalar and electromagnetic fields (in what follows, we shall follow book[3] in choice of notation):

$$(\partial_n - ieA_n)(\partial^n - ieA^n)\Psi = -m^2\Psi, \qquad (1)$$

$$\Box A_n + A^k_{,kn} = -j_n, \qquad (2)$$

$$j_n = -ie(\overset{*}{\Psi}\Psi_{,n} - \overset{*}{\Psi}_{,n}\Psi) - 2e^2 A_n \overset{*}{\Psi}\Psi. \qquad (3)$$

It was noted in work[2] that it is possible, using a gauge transform of the scalar and electromagnetic fields, to pass from equations (1) and (3) for A_n and Ψ to equations for B_n and φ, where φ is a real field:

$$\partial_n\partial^n\varphi = (e^2 B_n B^n - m^2)\varphi, \qquad (4)$$

$$j_n = -2e^2 B_n \varphi^2. \qquad (5)$$

Therewith, the currents j_n in equations (3) and (5) coincide, and in equation (2), A_n is replaced with B_n. Thus, the vast area of phenomena described by equations (1-3) may be described using the real field φ and the potential of electromagnetic field B_n, which is codirectional with the 4-vector of probability current density J_n. As was emphasized in[2], this fact testifies that to describe charged particles it is not necessary to introduce a complex field (or two real fields corresponding to the real and the imaginary part of a complex field). The results of work[2] suggest that spin 1/2 charged particles may be described using real spinors[4] (Majorana spinors[5]) only. Such suggestion was made, e.g., in work[6] (see, however, [7]). As is known, Majorana spinors are widely used to describe a massive spin 1/2 particle in connection with the mass of neutrino and neutrino oscillations[8,9], as well as in supersymmetric models of quantum field theory[10,11]. It should be noted that in this work non-quantized fields are considered, and therewith, in contrast with works[10,11], components of Majorana spinors are c-spinors, and not anti-commuting elements of Grassman algebra. Therefore, e.g., the zeroth component of the current vector vanishes for a zero spinor only.

The system of interacting electromagnetic and Majorana fields. Let us consider a system of equation of motion for interacting spinor and electromagnetic fields:

$$(i\hat{\partial} + e\hat{A} - m)\Psi = 0, \qquad (6)$$

$$\Box A_n + A^k_{,kn} = -j_n, \qquad (7)$$

$$j_n = -e\overline{\Psi}\gamma_n\Psi . \tag{8}$$

It is only possible to pass from the given spinor Ψ to a Majorana spinor, using a gauge transform, by analogy with work/2/, if the axial-vector current $\overline{\Psi}\gamma^n\gamma^5\Psi$ vanishes. Therefore let us seek such solutions only that Ψ is a Majorana spinor. Then equation (6) is equivalent to the following equalities:

$$(i\hat{\partial} - m)\Psi = 0 , \tag{9}$$

$$\hat{A}\Psi = 0 . \tag{10}$$

The fact that the spinor Ψ satisfies the free Dirac equation (9) does not mean that there is no interaction with electromagnetic field, as by far not all solutions of the free Dirac equation satisfy the constraint condition (10).

The system of equations (7-10) is overdetermined, but, as proven in work[/12/], has nontrivial solutions. If the vectors A_n and j_n do not vanish, then (10) is equivalent to[/12/]

$$j_n = \lambda A_n , \tag{11}$$

and, therefore, the probability current and the potential are codirectional, which is in accordance with the conclusions of works[/1,2/]. As for a Majorana spinor $j_n j^n = 0$, (11) implies

$$A_n A^n = 0 . \tag{12}$$

(12) may be regarded as a gauge condition. A somewhat more general condition $A_n A^n = const$ was considered in works[/1,13,14/].

(9) and (10) imply

$$\{\hat{\partial}, \hat{A}\}_+ \Psi = 2A^n\Psi_{,n} + A_{m,n}\gamma^n\gamma^m\Psi = 0 \tag{13}$$

and, therefore, the derivative in the direction A^n in a given point may be determined if only the value of Ψ in this point is known, and as the directions of the vectors of current and potential coincide, it is sufficient to know the value of Ψ in a given point to determine values of Ψ on the entire line of current passing through the given point.

Discussion. The system of equations of classical spinor electrodynamics (6-8), under the assumption that the spinor Ψ in equation (6) is a Majorana one, is thus equivalent to the following system:

$$\Box A_n + A^k_{,kn} = -\lambda A^k , \tag{14}$$

$$A_n A^n = 0 , \tag{15}$$

$$(i\hat{\partial} - m)\Psi = 0 , \tag{16}$$

$$j_n = -e\overline{\Psi}\gamma_n\Psi = \lambda A_n . \tag{17}$$

Equations of motion (14) correspond to the minimum of action for the Lagrangian of free electromagnetic field under condition (15) (λ is a Lagrange multiplier)[/1/], therefore, equations (14,15) describe independent propagation of electromagnetic field. Following work[/15/], one can associate with a Majorana spinor satisfying equations (16,17) an ensemble of point particles with space density $\overline{\Psi}\gamma_0\Psi$ moving along the lines of current (these properties will hold with time). One may assume that each particle from the ensemble is described by the spinor Ψ in a given point and moves independently in the field of the potential A_n in accordance with equation (13). The idea of such motion does not contradict the uncertainty principle as the instantaneous velocity of motion equals the velocity of light ($j^n j_n = A^n A_n = 0$), which formally corresponds to infinite energy and momentum and, therefore, to their complete uncertainty. Again, the lightlike instantaneous velocity is related to the well-known zitterbewegung[/16, p.342/], which is described by the Dirac equation. For example, for every solution of the Dirac equation with well-defined values of momentum and spin projection on the direction of momentum there exist corresponding Majorana spinors whose lines of current are spirals with the axis parallel to the momentum and the radius of the order of the Compton wavelength (cf./7/ and also/17,18/), which corresponds to motion with an instantaneous velocity equal to the velocity of light and average velocity connected with the momentum by the classical relation.

The role of the quantum potentials of work[/15/], whose physical meaning is not clear, is played in the outlined scheme by the 4-potential of electromagnetic field.

The use of Majorana spinors only does not mean departing from the description of antiparticles along with particles, as charge density for the Dirac equation is positive definite, and positrons do not correspond to states with negative energies, but to holes in the distribution of such states[/16, p.359/]. Therefore, one may assume that a Majorana spinor that is a solution of the Dirac equation simultaneously describes both an electron (as a particle above the background) and a positron (as the absence in the background of a particle in this state).

The limitation to Majorana spinors fixes the gauge condition (14) in the system (14-17). The equivalent gauge-invariant equations may be obtained by selecting not just Majorana solutions odf the system (6-8), but all solutions with vanishing axial-vector current (such solution may be reduced to Majorana ones by a gauge transform).

To establish the validity of equations (14-17) for description of the electron, it is necessary to resolve the following questions: is the set of potentials satisfying equations (14,15) rich enough? Do all such potentials have at least one Majorana spinor satisfying equations (16-17)? A positive reply to the first question (if the zero in the right-hand side of (15) is replaced with a small constant) was given in work[/1/]. A comparison with the results of work[/2/] suggests that the second question can also be answered in the affirmative, but a more rigorous treatment of this question is necessary.

<u>References.</u>

1. P.A.M.Dirac. Proc. Roy. Soc. London, <u>A209</u>, 291(1951)
2. E.Schrödinger. Nature, <u>169</u>, 538(1952)
3. N.N.Bogolubov, D.V.Shirkov. Introduction to the theory of quantized fields, 1976.
4. J.A.Schouten, D. van Dantzig. Z. Physik, <u>78</u>, 639(1932)
5. E.Majorana. Nuovo Cim., <u>14</u>, 171(1937)
6. V.N.Strel'tsov. On the probability current density for spin 1/2 density. Soobshch. OIYaI, R2-10155, 1976.
7. V.N.Strel'tsov. 4-vector of the probability current density of spinless particles and the Hamilton – Jacobi equation. Soobshch. OIYaI, R2-10930, 1977.
8. S.M.Bilenky, B.Pontecorvo. Phys. Rep., <u>41</u>, 225(1978)
9. P.H.Frampton, P.Vogel. Phys. Rep., <u>82</u>, 342(1982)
10. V.I.Ogievetskii, L.Mezinchesku. UFN, <u>117</u>, 637(1975)
11. A.Salam, J.Strathdee. Fortschr. Physik, <u>26</u>, 56(1978)
12. W.Buchmüller, K.Dietz, H.Römer. Phys. Lett., <u>64B</u>, 191(1976)
13. Y.Nambu. Suppl. Progr. Theor. Phys., Extra Number, 190(1968)
14. G.Venturi. Nuovo Cim., <u>65A</u>, 64(1981)
15. D.Bohm. Progr. Theor. Phys., <u>9</u>, 273(1953)
16. P.Dirac. Principles of quantum mechanics, 1979.
17. G.A.Perkins. Found. Phys., <u>6</u>, 237(1976)
18. G.A.Perkins. Found. Phys., <u>8</u>, 745(1978)